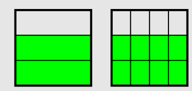
Grade 4

Grade 4 students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction. This property forms the basis for much of their other work in Grade 4, including the comparison, addition, and subtraction of fractions and the introduction of finite decimals.

Equivalent fractions Students can use area models and number line diagrams to reason about equivalence.[4.NF.1] They see that the numerical process of multiplying the numerator and denominator of a fraction by the same number, *n*, corresponds physically to partitioning each unit fraction piece into *n* smaller equal pieces. The whole is then partitioned into *n* times as many pieces, and there are *n* times as many smaller unit fraction pieces as in the original fraction.

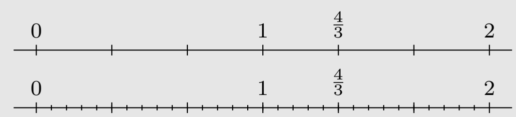
4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Using an area model to show that $\frac{2}{3} = \frac{4\times 2}{4\times 3}$



The whole is the square, measured by area. On the left it is divided horizontally into 3 rectangles of equal area, and the shaded region is 2 of these and so represents $\frac{2}{3}$. On the right it is divided into 4×3 small rectangles of equal area, and the shaded area comprises 4×2 of these, and so it represents $\frac{4\times 2}{4\times 3}$.

Using the number line to show that $\frac{4}{3} = \frac{5\times 4}{5\times 3}$



$\frac{4}{3}$ is 4 parts when each part is $\frac{1}{3}$, and we want to see that this is also 5×4 parts when each part is $\frac{1}{5\times 3}$. Divide each of the intervals of length $\frac{1}{3}$ into 5 parts of equal length. There are 5×3 parts of equal length in the unit interval, and $\frac{4}{3}$ is 5×4 of these. Therefore $\frac{4}{3} = \frac{5\times 4}{5\times 3} = \frac{20}{15}$.

This argument, once understood for a range of examples, can be seen as a general argument, working directly from the Grade 3 understanding of a fraction as a point on the number line.

The fundamental property can be presented in terms of division, as in, e.g.

$$\frac{28}{36} = \frac{28 \div 4}{36 \div 4} = \frac{7}{9}.$$

Because the equations $28 \div 4 = 7$ and $36 \div 4 = 9$ tell us that $28 = 4 \times 7$ and $36 = 4 \times 9$, this is the fundamental fact in disguise:

$$\frac{4 \times 7}{4 \times 9} = \frac{7}{9}.$$

It is possible to over-emphasize the importance of simplifying fractions in this way. There is no mathematical reason why fractions must be written in simplified form, although it may be convenient to do so in some cases.

Grade 4 students use their understanding of equivalent fractions to compare fractions with different numerators and different denominators.[4.NF.2] For example, to compare $\frac{5}{8}$ and $\frac{7}{12}$ they rewrite both fractions as

$$\frac{60}{96}\left(= \frac{12 \times 5}{12 \times 8}\right) \quad \text{and} \quad \frac{56}{96}\left(= \frac{7 \times 8}{12 \times 8}\right)$$

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, =, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Because $\frac{60}{96}$ and $\frac{56}{96}$ have the same denominator, students can compare them using Grade 3 methods and see that $\frac{56}{96}$ is smaller, so

$$\frac{7}{12} < \frac{5}{8}.$$

Students also reason using benchmarks such as $\frac{1}{2}$ and 1. For example, they see that $\frac{7}{8} < \frac{13}{12}$ because $\frac{7}{8}$ is less than 1 (and is therefore to the left of 1) but $\frac{13}{12}$ is greater than 1 (and is therefore to the right of 1).

Grade 5 students who have learned about fraction multiplication can see equivalence as "multiplying by 1":

$$\frac{7}{9} = \frac{7}{9} \times 1 = \frac{7}{9} \times \frac{4}{4} = \frac{28}{36}$$

However, although a useful mnemonic device, this does not constitute a valid argument at this grade, since students have not yet learned fraction multiplication.

Adding and subtracting fractions The meaning of addition is the same for both fractions and whole numbers, even though algorithms for calculating their sums can be different. Just as the sum of 4 and 7 can be seen as the length of the segment obtained by joining together two segments of lengths 4 and 7, so the sum of $\frac{2}{3}$ and $\frac{8}{5}$ can be seen as the length of the segment obtained joining together two segments of length $\frac{2}{3}$ and $\frac{8}{5}$. It is not necessary to know how much $\frac{2}{3} + \frac{8}{5}$ is exactly in order to know what the sum means. This is analogous to understanding 51×78 as 51 groups of 78, without necessarily knowing its exact value.

This simple understanding of addition as putting together allows students to see in a new light the way fractions are built up from unit fractions. The same representation that students used in Grade 4 to see a fraction as a point on the number line now allows them to see a fraction as a sum of unit fractions: just as $5 = 1+1+1+1+1$, so

$$\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

because $\frac{5}{3}$ is the total length of 5 copies of $\frac{1}{3}$.[4.NF.3]

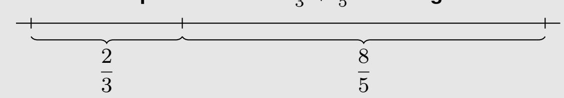
Armed with this insight, students decompose and compose fractions with the same denominator. They add fractions with the same denominator:[4.NF.3c]

$$\begin{aligned}\frac{7}{5} + \frac{4}{5} &= \overbrace{\frac{1}{5} + \cdots \frac{1}{5}}^{7} + \overbrace{\frac{1}{5} + \cdots \frac{1}{5}}^{4} \\ &= \frac{\overbrace{1 + 1 + \cdots + 1}^{7+4}}{5} \\ &= \frac{7+4}{5}.\end{aligned}$$

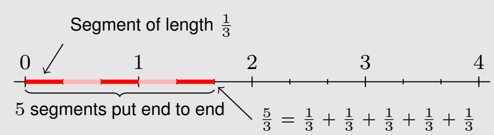
Using the understanding gained from work with whole numbers of the relationship between addition and subtraction, they also subtract fractions with the same denominator. For example, to subtract $\frac{5}{6}$ from $\frac{17}{6}$, they decompose

$$\frac{17}{6} = \frac{12}{6} + \frac{5}{6}, \quad \text{so} \quad \frac{17}{6} - \frac{5}{6} = \frac{17-5}{6} = \frac{12}{6} = 2.$$

Representation of $\frac{2}{3} + \frac{8}{5}$ as a length



Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$



4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

- a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
- c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Students also compute sums of whole numbers and fractions, by representing the whole number as an equivalent fraction with the same denominator as the fraction, e.g.

$$7\frac{1}{5} = 7 + \frac{1}{5} = \frac{35}{5} + \frac{1}{5} = \frac{36}{5}.$$

Students use this method to add mixed numbers with like denominators.• Converting a mixed number to a fraction should not be viewed as a separate technique to be learned by rote, but simply as a case of fraction addition.

• A mixed number is a whole number plus a fraction smaller than 1, written without the $+$ sign, e.g. $5\frac{3}{4}$ means $5+\frac{3}{4}$ and $7\frac{1}{5}$ means $7+\frac{1}{5}$.

Similarly, converting an improper fraction to a mixed number is a matter of decomposing the fraction into a sum of a whole number and a number less than 1.[4.NF.3b] Students can draw on their knowledge from Grade 3 of whole numbers as fractions. For example, knowing that $1 = \frac{3}{3}$, they see

$$\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}.$$

4.NF.3b Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Repeated reasoning with examples that gain in complexity leads to a general method involving the Grade 4 NBT skill of finding quotients and remainders.[4.NBT.6] For example,

$$\frac{47}{6} = \frac{(7\times 6)+5}{6} = \frac{7\times 6}{6} + \frac{5}{6} = 7 + \frac{5}{6} = 7\frac{5}{6}.$$

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

When solving word problems students learn to attend carefully to the underlying unit quantities. In order to formulate an equation of the form $A+B=C$ or $A-B=C$ for a word problem, the numbers A, B, and C must all refer to the same (or equivalent) wholes or unit amounts.[4.NF.3d] For example, students understand that the problem

> Bill had $\frac{2}{3}$ of a cup of juice. He drank $\frac{1}{2}$ of his juice. How much juice did Bill have left?

cannot be solved by subtracting $\frac{2}{3} - \frac{1}{2}$ because the $\frac{2}{3}$ refers to a cup of juice, but the $\frac{1}{2}$ refers to the amount of juice that Bill had, and not to a cup of juice. Similarly, in solving

> If $\frac{1}{4}$ of a garden is planted with daffodils, $\frac{1}{3}$ with tulips, and the rest with vegetables, what fraction of the garden is planted with flowers?

students understand that the sum $\frac{1}{3} + \frac{1}{4}$ tells them the fraction of the garden that was planted with flowers, but not the number of flowers that were planted.

4.NF.3d Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Multiplication of a fraction by a whole number Previously in Grade 3, students learned that 3×7 can be represented as the number of objects in 3 groups of 7 objects, and write this as $7+7+7$. Grade 4 students apply this understanding to fractions, seeing

$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} \quad \text{as} \quad 5\times\frac{1}{3}.$$